

ASIDE

①

```
do  
{  
  f x = _____  
  fprime x = _____  
  error =  
  value = value - error  
}  
while (abs(error) > tolerance):
```

② VARIABLE NAMES

tempF

tempK

A

B

$$\boxed{x_A = x_{Aeq}(1 - e^{-qt})} \quad \text{WHERE } q = \frac{k_1(\gamma + 1)}{(\gamma + x_{Aeq})}$$

ALSO RELATIONSHIPS, $x_{Aeq} = \frac{K_{eq} - \gamma}{K_{eq} + 1} \neq K_{eq} = \frac{\gamma + x_{Aeq}}{1 - x_{Aeq}}$

COMMENTS ① IF K_{eq} IS LARGE (ALMOST IRREVERSIBLE)



THEN $x_{Aeq} = \frac{K_{eq} - \gamma}{K_{eq} + 1} \longrightarrow$

①

AND IF $x_{Aeq} \approx 1$ THEN $q = \frac{k_1(\gamma + 1)}{\gamma + x_{Aeq}} \longrightarrow k_1$

INSERTING THESE TWO SIMPLIFICATIONS IN EQUATION ABOVE
YIELDS $x_A = 1 - e^{-qt}$ SAME RESULT AS FIRST ORDER IRREVERSIBLE.

AND $\chi_A \approx 1 - e^{-k_1 t}$... EXACTLY SAME RESULT
THAT WE OBTAINED
IN IRREVERSIBLE CASE!

② WE CAN USE THESE EQUATIONS IN 2 WAYS:

1A) GIVEN χ_{Aeq} FIND K_{eq}

1B) GIVEN K_{eq} FIND χ_{Aeq}

2A) GIVEN $K_{eq} \neq \chi_{Aeq}$ $\nearrow$ FIND χ_A AT A SPECIFIC TIME

2B) $\searrow$ FIND TIME TO REACH A SPECIFIC χ_A

EXAMPLE 4

THE POLLUTANT DESCRIBED IN EXAMPLE 3 ACTUALLY UNDERGOES A SLOW REVERSIBLE REACTION WITH A RATE CONSTANT (k_{-1}) OF $8.80 \times 10^{-5} \text{ DAYS}^{-1}$.



- A) HOW LONG WILL IT TAKE POLLUTANT ^("A") TO FALL BELOW A CONCENTRATION OF 5.0 ppb IF $C_{B0} = 0$?
- B) HOW LONG WILL IT TAKE POLLUTANT TO FALL BELOW A CONCENTRATION OF 5.0 ppb IF $C_{B0} = 1.0 \times 10^{-7} \text{ mol/L}$?
- C) IF $C_{B0} = 0$, WHAT IS THE LOWEST CONC. THAT THE POLLUTANT CAN ACHIEVE?

DATA FROM EXAMPLE 3:

$$k_1 = 5.20 \times 10^{-3} \text{ DAYS}^{-1}$$

$$C_{A0} = 8.453 \times 10^{-7} \text{ mol/L}$$

$$C_A(5.0 \text{ ppb}) = 0.274 \times 10^{-7} \text{ mol/L}$$

$$\text{ALSO } K_{EQ} = \frac{k_1}{k_{-1}} = \frac{5.20 \times 10^{-3} \text{ day}^{-1}}{8.80 \times 10^{-5} \text{ day}^{-1}} = 59.1$$

$$A) C_{B0} = 0 \quad Y = \frac{C_{B0}}{C_{A0}} = 0$$

$$\text{GIVEN } K_{EQ} \text{ CALCULATE } X_{A, EQ} = \frac{K_{EQ} - Y}{K_{EQ} + 1} = \frac{59.1}{60.1} = 0.9834$$

$$\rightarrow X_A = X_{A, EQ} (1 - e^{-qt}) \text{ will solve for } t$$

$$q = \frac{k_1(Y+1)}{Y + X_{A, EQ}} = \frac{(5.20 \times 10^{-3})(0+1)}{0 + 0.9834} = 5.288 \times 10^{-3} \text{ DAYS}^{-1}$$

AT
"5.0 ppb"

$$\chi_A = \frac{C_{A0} - C_A}{C_{A0}} = \frac{8.453 - 0.274}{8.453} = 0.9676$$

ASIDE

THE PROBLEM COULD BE RESTATED AS
"HOW LONG WILL IT TAKE FOR χ_A TO REACH 0.9676
GIVEN THAT THE EQUILIBRIUM χ_{Aeq} IS 0.9834"

$$0.9676 = 0.9834 (1 - e^{-5.288 \times 10^{-3} t})$$

$$\underline{t = 781 \text{ DAYS}}$$

(COMPARE WITH
ANSWER
IN EXAMPLE 3)

B) $C_{B0} = 10.0 \times 10^{-7} \text{ mol/L}$



WE HAVE TO ASK WHAT IS THE INITIAL CONC.
OF "A" BEFORE THE SPILL?

ASSUME THAT "A" IS IN EQUILIBRIUM WITH "B"
PRIOR TO SPILL. THE SPILL JUST ADDS TO THAT "A".

BEFORE SPILL

$$K_{EQ} = 59.1 = \frac{C_{B_{EQ}}}{C_{A_{EQ}}} = \frac{10.0 \times 10^{-7}}{C_{A_{EQ}}} \Rightarrow C_{A_{EQ}} = 0.169 \times 10^{-7} \text{ mol/L}$$

AFTER SPILL

$$C_{A0} = 8.453 + 0.169 = 8.622 \times 10^{-7} \text{ mol/L}$$

SAME PROCEDURE AS IN PART A)

$$\gamma = \frac{C_{B0}}{C_{A0}} = \frac{10}{8.622} = 1.16$$

$$\chi_{Aeq} = \frac{K_{eq} - \gamma}{K_{eq} + 1} = \frac{59.1 - 1.16}{59.1 + 1} = 0.9641$$

$$\tau = \frac{k_1 (\gamma + 1)}{(\gamma + \chi_{Aeq})} = \frac{(5.20 \times 10^{-3}) (2.16)}{(1.16 + 0.9641)} = 5.288 \times 10^{-3} \text{ days}^{-1}$$

$$\chi_A = \frac{8.622 - 0.274}{8.622} = 0.9682$$

ASIDE

HOW LONG WILL IT TAKE χ_A TO REACH 0.9682
GIVEN THAT THE EQUILIBRIUM χ_{Aeq} IS 0.9641?